Final Performance Report

NONLINEAR CONTROL SYSTEMS

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CHAPTER I

EXECUTIVE SUMMARY

The principal goal of this research program was to develop a systematic methodology for the design of feedback control schemes capable of shaping the response of complex dynamical systems. A continuation of an ongoing research effort, the specific research program we proposed was aimed at the development of a systematic control methodology for lumped and distributed parameter systems, applicable to both the equilibrium and the nonequilibrium cases. The typical design objectives involve designing feedback schemes which achieve one or more of the following: asymptotic tracking, an appropriate form of internal stabilization, and asymptotic disturbance rejection.

For linear and nonlinear systems, stability has been classically defined as closed-loop exponential stability of an equilibrium. In the equilibrium case, when taken together these form the control task classically known as the *servomechanism* or *output regulation* problem, a problem which is one of the defining problems of classical automatic control. For linear multivariable systems this problem was addressed in very elegant geometric terms by Davison, Francis, Wonham [26, 28] and others. In particular, one of the most relevant contributions of [28] was a clear delineation of what is known as *internal model principle*, i.e. the fact that the output regulation property is insensitive to plant parameter variations "only if the controller utilizes feedback of the regulated variable, and incorporates in the feedback path a suitably reduplicated model of the dynamic structnre of the exogenous signals which the regulator is required to process." Conversely, in a stable-closed loop system, if the controller utilizes feedback of the regulated variable and incorporates an internal model of the exogenous signals, the output regulation property is insensitive to plant parameter variations.

There is now an extensive literature on output regulation for nonlinear control systems. As we describe, part of our ongoing research effort is the design of internal models, using immersion techniques, for attenuating disturbances and tracking reference trajectories for nonlinear control systems. This effort was transitioned by Boeing to the development of improved (e.g., lower cost, lower weight) actuators for UCAV's, a transition which has been tested with success using their

X-45A simulator.

In general, there are important existing and emerging DOD research and development programs which will require nonequilibrium control methodologies. While flight control in a trim condition is a typical problem of output regulation near an equilibrium setting, tailless or nearly tailless aircraft, such as UCAV's, will also have multiple nonlinear axes and a smaller domain of stability than conventional aircraft, involving nonlinear trajectories which cannot be regarded as small perturbations of an asymptotically stable equilibrium. Moreover, these aircraft enjoy less drag and more agility, and require less control to produce significant nonlinear excursions. Taking advantage of these nonequilibrium nonlinearities in tailless aircraft also promises to impact affordability by enabling the use of smaller, low cost actuators to enhance agility and maneuverability.

As another motivation, the design of aircraft capable of longer strike capabilities has become more important as access to a worldwide network of air bases diminishes,. These aircraft will typically need to weigh less and have increased agility. Both of these desirable characteristics will need controller designs which are able to tolerate, or even incorporate, significant nonlinear excursions. As another example, the JDAM kit, which can convert conventional missiles into smart weapons, induces a periodic, spinning, motion during the end game in order to improve accuracy as well as to acquire additional information about the target. Underscoring the nonlinear, nonequilibrium nature of this problem is the very existence of (locally) isolated, stable periodic orbits, a nonlinear phenomenon which has no linear counterpart. Such examples also point to an important feature of nonequilbrium controller design which is desirable for existing and emerging DoD technologies, the potential to take advantage of - rather than to attenuate - nonlinear effects.

Of course, one of the essential features of being able to shape the steady-state response of a nonlinear feedback system is to clearly delineate what the steady-state response of a control system is. To this end, we proposed developing this concept of steady-state response for input-output feedback systems that typically arise in in problems of non-equilibrium nonlinear feedback systems. After considerable research, we have accomplished this research task, using the notion of the limit set of a set, rather than the more customary concept of the limit set of a point, pioneered by G. D. Birkhoff. This broader concept was used by Ladyzhenskaya in her study of the two dimen-

sional Navier-Stokes equation and by Hale and Sell in their study of general infinite dimensional dynamical systems.

As discussed above, another key to output regulation is the design and use of internal models to yield asymptotic proxies for state feedback laws which would be classically designed using knowledge of states and uncertain parameters, but which can, in fact, be unobservable from the measured output. Our current design philosophy is the parallel design of two separate controllers. The first controller, commonly referred to as the *internal model*, generates an asymptotic proxy for the input which renders invariant a set on which the error variables are zero. Under an appropriate system invertibility hypothesis, this input is unique and is typically a function of state variables and uncertain parameters. For this reason, the internal model contains an immersed copy of the exosystem and a set of uncertain parameters but is itself detectable using a proxy for the desired input as the output of the internal model. In the classical local case, we have shown that the second system locally exponentially stabilizes the interconnection of the plant and the first controller. In the nonequilibrium case, it renders the invariant set attractive.

The other focus of our proposed research program focused on the problem of output regulation for linear and nonlinear distributed parameter systems. The incorporation of linear and nonlinear distributed parameter effects also presents an opportunity in the control of complex dynamical systems. Indeed, flow control provides examples of the potential impact of nonlinear control of distributed parameter systems involving both equilibrium and nonequilibrium cases. For example, active control of flutter and of buffeting would increase the life-cycle of aircraft through suppressing buffet loads on fighter aircraft with vertical tails as well as on the tails of commercial and transport aircraft. Our research efforts focused on developing a systematic design methodology for the output regulation of linear parabolic boundary control systems in n spatial dimensions. For set-point control problems for lumped systems or for distributed parameter systems evolving in one spatial dimension, the exosystem generating constant signals is finite dimensional. In higher spatial dimensions, however, it often will need to be infinite dimensional and we needed to begin with an extension of the more classical cases to this setting. We proposed two approaches to this research task. The first was based on the geometric theory of the regulator equations, and has been successfully researched for bounded input and output operators in the literature. In this report, we also describe our research in the unbounded case. The second approach is based on compensator

design using zero dynamics, ultimately leading to an infinite dimensional compensator.

Another of our longer term goals was the development of a theory of nonlinear output regulation as parallel as possible to the theory we envision for linear problems. While output regulation is an asymptotic theory and the long time existence of solutions to open-loop nonlinear distributed parameter systems remains extremely challenging, we have been successful in establishing long time existence and asymptotic behavior for the control of certain examples or system classes using particular feedback design methods ([20, 21, 16, 17, 19, 18]). Still, the control of nonlinear distributed parameter systems is sufficiently difficult that our proposed research efforts have focused on local results for output regulation with respect to signals and disturbances generated by finite-dimensional exogeneous systems, where techniques such as center manifold methods can yield some powerful insights. We emphasize the fact that these local techniques are not simply an appeal to linearization. Even in the lumped nonlinear case, elementary examples show that a solution to the problem of output regulation for the linearization does not solve the output regulation problem for the nonlinear problem.

CHAPTER II

RESEARCH TASKS AND ACCOMPLISHMENTS

2.1 Output regulation for lumped nonlinear systems

An essential aspect of output regulation, in both the equilibrium and the nonequilibrium cases, is the development of a model for a system which generates the disturbances to be rejected or the signals to be tracked. The generators of these two types of signals can be connected in parallel, so that we typically assume there is one exogenous signal generator. One of our objectives was to investigate the properties of exogenous signal generators, as well as to delineate the properties of bounded signals which can be generated by exogenous systems with an appropriate form of stability. The goals of this research thrust were described in the body of our proposal. In the classical equilibrium approach to output regulation, in order to produce periodic exogenous signals one is forced into the unecessary compromise of using an exosystem with an equilibirium, such as the harmonic oscillator. As an extreme example, in the nonlinear case, every perioidic singal, with a given period, is some nonlinear output of the one-dimensional system, $\dot{\tau} = 1$. In our research we have considerably enhanced classical output regulation theory by including nonlinear exosystems with no equilibria. Another of our objectives was to develop the foundations for a non-equilibrium theory of nonlinear output regulation, giving a more general (non-equilibrium) definition of the problem.

Of course, one of the essential features of being able to shape the steady-state response of a nonlinear feedback system is to clearly delineate what the steady-state response of a control system is. After considerable research, we have accomplished this research task, using the notion of an ω-limit set of a set, rather than the more customary concept of an ω-limit set of a point, pioneered by G. D. Birkhoff. This hroader concept was used by Ladyzhenskaya in her study of the two dimensional Navier-Stokes equation and by Hale and Sell in their study of general infinite dimensional dynamical systems. In [7], we developed this concept for input-output feedback systems that typically arise in in problems of non-equilibrium nonlinear feedback systems.

This recent advance is significant and clarifies and extends our previous research on a non-equilibrium theory of nonlinear output regulation. In particular, it allows for a more general (non-equilibrium) definition of the problem, deriving necessary conditions, and, using these necessary

conditions, we present a set of sufficient conditions and a design methodology for the solution of the problem in question. Our analysis leads to a non-equilibrium enhancement of the internal model principle, which can be expressed as a relationships between two uniformly stable attractors. The first is an attractor for the combined dynamics of the exogenous signal generator and the so-called zero-dynamics of the plant to be controlled, intrinsic to the formulation of the problem. The second is the uniformly stable attractor for the dynamics of the closed-loop system determined by the controller which solves the problem of output regulation, under hypotheses which are non-equilibrium enhancements of those familiar from the equilibrium case. This enhancement of the internal model principle asserts, roughly speaking, that any controller solving the problem of output regulation has to contain a copy of an attractor which may combine the dynamics of the exogenous system with certain nontrivial steady-state motions occurring in the plant to be controlled. In the simple case in which there is only one (and trivial) such steady-state motions, and the analysis is only local, the theory we develop reduces to the one presented in our earlier work. On the other hand, the more general approach discussed here makes it possible to solve problems to which none of the existing design methods for output regulation is applicable.

The foundations of this theory were originally presented in [24], which relied heavily on [33] in a preliminary preprint form. For the sake of completeness we shall review the basic assumptions considered in this work. Rather than assuming that the zero-dynamics of the controlled plant have a globally asymptotically stable equilibrium, this assumption is replaced with the (substantially weaker) hypothesis that the zero dynamics of the plant "augmented by the exsosystem" have a compact attractor. In this work, though, we have retained the (rather strong) assumption, itself also common to all earlier literature, that the set of all "feedforward inputs capable of securing perfect tracking" is a subset of the set of solutions of a suitable *linear* differential equation (assumption of "immersion" into a linear system). In the subsequent paper [25] we showed that, within the new framework, the assumption of linearity can also be weakened and replaced by the assumption that the set in question is a a subset of the set of solutions of a suitable *nonlinear* differential equation (assumption of "immersion" into a nonlinear system). These results were subsequently generalized to the case of a system having higher relative degree, by showing how output regulation can be achieved by means of a (dynamic) pure error feedback.

2.2 Nonlinear oscillations

The most classical example of a nonequilibrium attractor for a nonlinear dynamical system is a periodic orbit. In two dimensions, Poincaré-Bendixson Theory gives a complete criterion for the existence of periodic orbits for differential equations leaving a bounded domain invariant and having no equilibria in the domain. The only bounded planar domain with these properties is an annulus. It has long been a goal in the theory of dynamical systems and ordinary differential equations to extend some versions of Poincaré-Bendixson Theory to higher dimensions, and some progress has been made some on replicating this theory, under fairly strong hypotheses. Most of the research starts with a region, generalizing the two dimensional annulus, that is positively invariant. More explicitly, suppose $M \subset \mathbb{R}^3$ is a closed bounded domain with smooth boundary which is diffeomorphic to $\mathbb{D}^{n-1} \times S^1$. We say that M, which is shaped like a higher dimensional "doughnut," is a solid n-torus. For example, it is a famous question of Smale [11] as to whether every nowhere zero vector field that points inward on the boundary of a solid three torus must have a periodic orbit. If this had turned out to be true, it would have been a neat generalization of an essential part of the Poincaré-Bendixson theory to three dimensions. Moreover, in three and higher dimensions, there are a lot of examples of nonlinear systems with this property arising in biology, chemistry, physics and engineering [8]. As it turns out, the answer to Smale's question is *no*.

Nonetheless, in [2] we have developed a positive answer to a more restricted set of such problems by incorporating some ideas similar to those in Lyapunov theory, but adapted for the ease of periodic orbits rather than for equilibria. In some more detail, an angular one-form on a bounded domain D for a vector field X in $\mathrm{Vect}(\mathbb{R}^n)$ is a closed differential one-form $\omega = \sum_{i=1}^{n} a_i dx_i$ such that

$$\langle \omega, X \rangle = \sum_{i=1}^{n} a_i X_i > 0$$

where X_i, for $i = 1, \ldots, n$ are the components of the vector field X.

In [2] we have proved the following result.

Theorem 2.1. *Suppose $X \in \mathrm{Vect}(\mathbb{R}^n)$ defines a differential equation $\dot{x} = f(x)$ on $\mathbb{R}^n$ which leaves a solid torus M positively invariant. If X has an angular one-form, then X has a periodic solution.*

Moreover, in [2] we have proved that the conditions hypothesized in this theorem are necessary conditions for the existence of a locally asympotically periodic orbit for a smooth vector field X on any smooth manifold, including $\mathbb{R}^n$.

Our most recent research directions include the continuing development of a more general, user-friendly criterion [1] based on the results of [2] as well as its applications to specific differential equations, such as those describing biological and electronic systems. In particular, we are developing a general criterion that applies to specific differential equations, such as those describing a phase-locked loop circuit and mathematical models of biological systems, such as the May-Leonard equations describing the population dynamics of three competing species with immigration into the population pool.

2.3 Asymptotic proxies and moment problems for signals, systems and control

In this project, we also proposed to research a general concept embodying the behavior of asymptotic proxies for a system state or for state feedback laws. This was based on a serendipitous discovery we have made as we tried to glean some insight about asymptotic proxies by analyzing the dynamics of a fast filtering algorithm, viewed as a nonlinear, discrete-time dynamical system on the space of positive real transfer functions. Our original interest in these dynamics stemmed from the asymptotic evolution of proxies for the Kalman gain, but the analysis of the phase portrait also contributed to the solution of two long-outstanding problems in interpolation theory.

The first resulted in a new design methodology for speech analysis and synthesis, including speaker recognition as a biometric and a new technique for high-resolution spectral estimation. Four U.S. Patents have now been granted for these new design methodologies. The development of this phase portrait also contributed to the solution of the rational Nevanlinna-Pick interpolation problem with degree constraints, leading to a new methodology for shaping the response of robust control systems. Indeed, many robust control problems, including the standard H^∞ problem, can be reduced to Nevanlinna-Pick interpolation. This approach yields a systems theoretic parameterization, in the discrete-time case, of all solutions not exceeding the degree of the widely used central solution. In addition, we have shown that each such solution is the unique minimum of a convex, mixed entropy integral that can be derived as the primitive of a closed one-form which is canonically associated to the generalized moment formulation of the Nevanlinna-Pick interpolation

problem. The moment problem as formulated by Krein and Nudel'man is a beautiful generalization of several important classical moment problems, including the power moment problem, the trigonometric moment problem and the moment problem arising in Nevanlinna-Pick interpolation. However, the importance of rational functions in systems and control and other engineering applications imposes certain complexity constraints. In [3, 5, 4] we present a new formulation of the moment problem which respects these constraints. While this version of the problem is decidedly nonlinear, the basic tools still rely on convexity. We give a complete parameterization of all solutions. This can be seen as a global analysis approach, where one studies an entire class of solutions as a whole. We then show that each solution in this class can be obtained by minimizing a strictly convex nonlinear functional. Thus the methodology employed is a combination of nonlinear analysis, geometry and optimization. Finally, we apply these results to interpolation problems of the Carathodory and of the Nevanlinna-Pick type, arising in signal processing and control theory, where we consider smooth bijective transformations from spaces of tuning parameters to entire classes of solutions.

Prediction-error methods for ARMA modeling play a major role in system identification, but in general they lead to nonconvex optimization problems for which global convergence is not guaranteed. In fact, although these algorithms are computationally simple and quite reliable, there is so far no theoretically satisfactory algorithm for ARMA parameter estimation. In [10] we identify certain classes of ARMA models in which prediction error minimization leads to convex optimization. It is known that model approximation via prediction error identification leads to an optimization problem that is related to the minimization of the Kullback-Leibler divergence criterion. This, in turn, leads naturally to our theory of analytic interpolation and generalized moment problems with complexity constraints.

In [C. I. Byrnes, P. Enqvist, and A. Lindquist, Identifiability and well-posedness of shaping-filter parameterizations: A global analysis approach, SIAM J. Control and Optimization, **41** (2002), 23–59] we proved that any stochastic process of degree n is characterized by its first $n+1$ covariance coefficients together with its first n cepstral coefficients. This paper also contains a solution of the corresponding inverse problem. In [9] we provide an approximation-theoretic justification of this solution. More precisely, we approximate any given spectral density with a rational one having a prescribed number of poles and zeros (n poles and m zeros inside the unit disc and their

conjugates), as a model-matching problem of a type previously considered by other authors in the context of least-squares estimation. Here we instead utilize the Kullback-Leibler divergence as a distance measure. The stationarity condition for optimality requires that the approximant matches $n+1$ covariance moments of the given power spectrum and m cepstral moments of the corresponding logarithm, although the latter with possible slack.

Over the past three decades there has been interest in using Padé approximants K with $n = deg(K) < deg(G) = N$ as reduced-order models for the transfer function G of a linear system. The attractive feature of this approach is that by matching the moments of G we can reproduce the steady-state behavior of G by the steady-state behavior of K, for certain classes of inputs. Indeed, in [6] we illustrate this by finding a first-order model matching a fixed set of moments for G, the causal inverse of a heat equation. A key feature of this example is that the heat equation is a minimum phase system, so that its inverse system has a stable transfer function G and that K can also be chosen to be stable. On the other hand, elementary examples show that both stability and instability can occur in reduced order models of a stable system obtained by matching moments using Padé approximants and, in the absence of stability, it does not make much sense to talk about steady-state responses nor does it make sense to match moments. In this paper, we review Padé approximants, and their intimate relationship to continued fractions and Riccati equations, in a historical context that underscores why Padé approximation, as useful as it is, is not an approximation in any sense that reflects stability. Our main results on stability and instability states that if $N \geq 2$ and $l, r \geq 0$ with $0 < l+r = n < N$ there is a non-empty open set of stable transfer functions G, having infinite Lebesque measure, such that each degree n proper rational function K matching the moments of G has l poles lying in C^- and r poles lying in C^+. The proof is constructive.

2.4 Sensitivity, Bifurcation and Uniqueness for Hydrodynamic Systems

Of considerable importance in fluid dynamics are problems related to predicting the onset of turbulence and understanding the underlying mechanisms. During this research period, in a joint work with John Burns at VPI & State University, we have made some interesting discoveries related to these issues for a model problem described by a viscous Burgers' equation. In particular, consider the initial boundary value problem for the one dimensional Burger's equation with

Reynolds number R

$$z_t(x,t) = R^{-1} z_{xx}(x,t) - z(x,t) z_x(x,t),\ 0 \le x \le 1, \tag{2.1}$$

$$z(x,0) = \varphi(x)$$

$$z_x(0,t) = 0,$$

$$z(1,t) = 0,$$

It can be rigorously proven (see e.g., [69]) that for every initial condition φ the time dependent solution $z(x,t)$ will approach zero, i.e.,

$$\lim_{t\to\infty} z(x,t) = 0 \quad \text{for all} \quad 0 \le x \le 1.$$

Nevertheless, one finds that depending on the size and shape of the initial condition the numerical solution of this problem (using any standard numerical method) instead converges to a nonzero steady state solution. In [67] we were able to show that, for a finite difference time marching scheme (based on the Crank–Nicolson method) this strange anomaly is due to the particular nonlinearity in Burgers equations and the fact that all computers use finite floating point arithmetic. Indeed, for any given computer arithmetic we can predict exact conditions on an initial condition for which the solution will converge to a false (nonzero) solution.

In continued efforts in studying this example we have exhibited in numerical simulations that this situation can be explained in terms of a problem in sensitivity of the Neumann boundary condition to a small non-homogeneous term. Namely we have been able to show in numerical experiment that the false solutions arise as solutions to a bifurcation problem that derive from (2.1) by replacing the homogeneous boundary condition at $x = 0$ by a non-homogeneous coudition. In particular these false solutions are solutions of the problem

$$z_t(x,t) = R^{-1} z_{xx}(x,t) - z(x,t) z_x(x,t),\ 0 \le x \le 1, \tag{2.2}$$

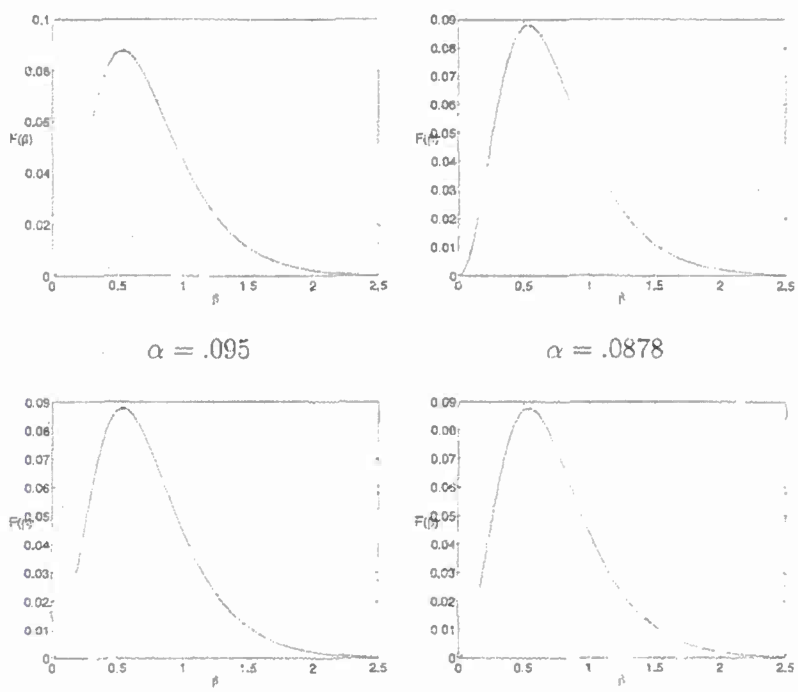
$$z(x,0) = \varphi(x)$$

$$z_x(0,t) = -\alpha,$$

$$z(1,t) = 0,$$

where α is a small (bifurcation) parameter. Indeed, even for positive nnmbers α less than machine precision zero we can, depending on the initial condition, obtain a nonzero steady state.

With the Reynolds number R and the (constant) non-homogenous Neumann boundary input α considered as bifurcation parameters we first show that, for each fixed R there is a saddle-node bifurcation with respect to the boundary parameter α. Namely, for large α the associated stationary Burgers' problem has no equilibria but for decreasing α there is a critical value at which there is a single stationary solution which has the following property: The linearization of the spatial Burgers' operator about this stationary solution has a simple zero eigenvalue and all others are negative. For all smaller values of α, there are two stationary solutions, denoted by $h_L(x)$ (Left) and $h_R(x)$ (Right).



$\alpha = .095$ $\qquad$ $\alpha = .0878$

$\alpha = .08$ $\qquad$ $\alpha = .04$

Fig.1 Bifurcation in α for fixed R

The stationary solution $h_L(x)$ is stable and generally very small (near zero) while the stationary solution $h_R(x)$ is (very slightly) unstable and tends to be large in comparison with $h_L(x)$. The linearization about $h_R(x)$ has one very small positive eigenvalue and all other eigenvalues are neg-

ative. Nevertheless, for small values of α (and larger values of R) an interesting anomaly appears, for certain initial data the solution of the corresponding time dependent Burgers' problem can converge to a spurious (nonexistent) numerical stationary solution very near $h_R(x)$. We expect that this situation may well be related to the general problem of sensitivity and onset of turbulence. We note that, in a standard way, this problem can be recast as a somewhat different bifurcation problem for Burger's equation with a non-homogenous forcing term. In a related work we have obtained a complete bifurcation analysis of homogeneous and non-homogeneous boundary controlled viscous Burgers' equation in [12]. We expect that the results in [12] will be useful in obtaining a complete understanding of the bifurcation mechanisms for the boundary controlled Burgers equation and, in turn, we expect this to shed new light on the corresponding problems in hydrodynamics.

The existence of such false solutions is very disturbing. This is especially true considering that a control mechanism based, for example, on a controller using such a model could produce devastating results.

More recent work by the authors strongly suggests that a similar anomaly takes place for a broad class of nonlinear parabolic equations containing convective type terms. Moreover, there is a strong numerical evidence that the same type of anomaly may occur for real hydrodynamic equations – Euler and Navier Stokes. We have identified a sensitivity/bifurcation phenomenon which, together with unavoidable limitations of computer arithmetic, has lead us to understand, at a fundamental level, the underlying mathematical origins and of strictly numerical solutions to certain hydrodynamic type boundary value problems. We expect that similar such bifurcations occur for many similar problems and therefore that one must include such considerations in any computational study of non-unique solutions to partial differential equations that govern physical systems such as fluid flows.

2.5 Geometric Output Regulation for Nonlinear Distributed Parameter Systems

Our second main area of research in distributed parameter systems was concerned with pursuing our systematic efforts toward the design of control laws for problems of tracking and disturbance rejection, i.e., regulation, for a general class of linear and nonlinear distributed parameter systems for a wide range of applicable control and sensing mechanisms. In this area we have significantly extended our earlier work on design methods based on geometric constructs involving

the regulator equations to include boundary controlled nonlinear distributed parameter systems. Here we have studied the problem of solving or approximating solutions to the regulator equations using iterative methods such as Newton iteration and various fixed point methods in infinite dimensional Banach spaces.

An important goal in the development of a systematic theory of nonlinear output regulation is to establish a theory as parallel as possible that which has been established for finite dimensional linear [26, 27, 28] and nonlinear systems ([33, 32, 13, 30, 34]). In this direction, we note that for a large class of linear DPS problems those state feedback control laws which solve the problem of output regulation for a stable linear system with bounded inputs and outpnts can also be characterized in an appealing systems theoretic fashion [36, 37], [38, 39] and [18].

Output regulation is an asymptotic theory and the long time existence of solutions to open-loop nonlinear distributed parameter systems remains extremely challenging. Nonetheless, we have been successful in establishing long time existence and asymptotic behavior for certain examples or system classes using particular feedback design methods (see, e.g., [20, 21, 16, 17, 19, 18]). For example, our current efforts are primarily focused on local results for output regulation with respect to signals and disturbances generated by finite-dimensional exogeneous systems (see, however, [31] for a discussion of infinite-dimensional exosystems). In our setting, the exosystem is both finite dimensional and *neutrally stable* [33] and we can appeal to powerful center manifold methods to obtain some nontrivial insights and results. We emphasize the fact that these local techniques are not simply an appeal to linearization. Even in the lumped nonlinear case, elementary examples [23] show that a solution to the problem of output regulation for the linearization does not solve the output regulation problem for the nonlinear problem.

Thus we have considered the output regulation problem for a special class of nonlinear distribnted parameter systems (NLDPS). The main goal of our work was to show that the geometric theory of nonlinear output regulation, which has been extensively developed for lumped nonlinear systems, can be extended in a local setting to this class of NLDPS. Our approach is geometric, based on the center manifold theorem. Even for local problems, however, one must surmount technical issues that inevitably arise in the infinite dimensional setting. The particular class of nonlinear systems and exogenous systems for which center manifold methods can be used to obtain state feedback control laws for solving problems of tracking and disturbance attenuation is

quite large and includes most common systems arising in the literature. The main restriction in our main result is that the input and output operators are bounded in the Hilbert state space. This means that things like point observations are not allowed. This is merely a technical issue and providing detailed proofs in these more general cases is currently in progress.

Thus consider an abstract nonlinear infinite dimensional system

$$\dot{z} = Az + f(z) + Bu + d \tag{2.3}$$

$$\dot{w} = s(w) \tag{2.4}$$

$$z(0) = z_0, \tag{2.5}$$

$$y = c(z), \quad y_r = q(w) \tag{2.6}$$

$$e = y - y_r.$$

where z is the state of the system in the infinite dimensional Hilbert space $\mathcal{Z}$; u is a control; y is the output and $z_0 \in \mathcal{Z}$ is the initial state of the system and d is a disturbance. Here the nonlinear term $f(z)$ satisfies $f(0) = 0$, $f_z(0) = 0$. Conditions on the linear operator A are given below.

In addition we assume there exists a *neutrally stable* [33] finite dimensional exogenous system

$$\frac{dw}{dt} = s(w) \tag{2.7}$$

$$w(0) = w_0 \in \mathcal{W}, \tag{2.8}$$

(here we assume that $\mathcal{W}$ is a finite dimensional Hilbert vector space) that generates both a reference signal y_r and the disturbance d. Namely, we assume

$$y_r(t) = q(w(t)) \quad q : \mathcal{W} \mapsto Y. \tag{2.9}$$

$$d(t) = p(w(t)) \quad p : W \mapsto Z. \tag{2.10}$$

The objective of output regulation is to find a control law

$$u = \gamma(w) = \Gamma w + \widetilde{\gamma}(w),$$

$$\Gamma \in \mathcal{L}(\mathcal{W}, \mathcal{U}), \;\; \widetilde{\gamma}(0) = 0, \;\; \frac{\partial \widetilde{\gamma}}{\partial w}(0) = 0.$$

so that closed-loop trajectories exist and so that the error

$$e(t) = y(t) - y_r(t) = c(z(t)) - q(z(t)).$$

exists as $t \longrightarrow +\infty$ and tends to 0.

Assumption 2.5.1. 1. $(-A)$ is a sectorial operator with compact resolvent. Therefore $(-A)$ generate a Hilbert scale Z_α.

2. The analytic semigroup $T(t) = e^{At}$ is exponentially stable (Notice that it is also a contraction semigroup).

3. $B \in \mathcal{B}(\mathcal{U}, \mathcal{Z})$ and $P \in \mathcal{L}(\mathcal{W}, \mathcal{Z})$ are bounded.

4. $C \in \mathcal{B}(Z_\alpha, Y)$ for some $\alpha > 0$, i.e., there is a constant c_α so that $\|C\varphi\|_y \leq c_\alpha \|\varphi\|_\alpha$.

5. We assume that the exosystem has the origin as a neutrally stable equilibrium, i.e., $w = 0$ is a fixed point which is Lyapunov stable but not attracting. A center is an example of such a fixed point. This, in particular, implies $\sigma(S) \subset i\mathbb{R}$ (i.e., the spectrum of S is on the imaginary axis) and has no non-trivial Jordan blocks.

Remark 2.1. In our examples, and quite often in practice, A is self-adjoint. We also note that, since we assume $(-A)$ is sectorial, we work with the semigroup $\exp(At)$ rather than $\exp(-At)$ as is done, for example, in Henry [29].

In order to simplify the exposition we will impose the following simplifying assumptions.

Assumption 2.5.2. *For simplicity of the exposition we will assume that the input and measured output are linear functions of the state of the plant and reference signal and disturbance are linear functions of the state of the exosystem. Thus we assume*

$$c(w) = Cw, \; q(w) = Qw.$$

The following result was obtained in [14].

Theorem 2.1. *Under assumptions 2.5.1 and 2.5.2, the (local) state feedback regulator problem for* (2.3)-(2.6) *is solvable if, and only if, there exist mappings* $\pi : \mathcal{W} \to D(A) \subset \mathcal{Z}$ *and* $\gamma : \mathcal{W} \to \mathcal{Y}$ satisfying the "regulator equations,"

$$\frac{\partial \pi}{\partial w} s(w) = A\pi(w) + f(\pi(w)) + B\gamma(w) + Pw \tag{2.11}$$

$$c(\pi(w)) = q(w). \tag{2.12}$$

In this case a feedback law solving the state feedback regulator problem is given by

$$u(t) = \gamma(w)(t). \tag{2.13}$$

Modulo the inherent technical difficulties that arise in infinite dimensions, Theorem 2.1 can be obtained using an argument similar to that given in [33]. Indeed, under the assumptions on A, B and C, we can appeal to a version of the Center Manifold Theorem to aid in the proof.

Not every problem of output regulation is solvable. Indeed even in the linear case there is a non-resonance condition requiring that the transfer function of the plant

$$G(s) = C(sI - A)^{-1}B, \ \ \sigma \in \rho(A), \tag{2.14}$$

be left invertible for all s in the spectrum of S, i.e., no eigenvalue of the exo-system, $s \in \sigma(S)$, is a transmission zero of the plant.

Assuming that the regulator equations are solvable in [15] we considered the problem of obtaining approximate solutions π and γ of the regulator equations (2.11), (2.12). In the typical parabolic case, A is an elliptic operator and the first regulator equation produces a system of nonlinear elliptic boundary value problems. This elliptic system is then coupled with an auxiliary operator equation via the constraint in the second regulator equation.

The approximate methods proposed here are based on Newton or fixed point iteration. To this end we introduce the following notation for the decomposition of the solutions π and γ into the linear parts which solve the regulator problem for the corresponding linear problem. Namely, set

$$\pi(w) = \Pi w + \widetilde{\pi}(w), \quad \gamma(w) = \Gamma w + \widetilde{\gamma}(w).$$

Here Π and Γ solve the linear approximation to the regulator equations in (2.11), (2.12)

$$\Pi S w = A\Pi w + (B\Gamma + P)w, \tag{2.15}$$

$$C\Pi w = Qw, \ \ \forall \ w \in \mathcal{W}.$$

Under our assumptions on A and S it is straightforward to obtain simply representations for Π and Γ and we can therefore consider small perturbations of the solutions [38, 39, 36, 19, 18] for the linear problem. Using the above notations we can write the regulator equations as

$$\left(\Pi w + \frac{\partial \widetilde{\pi}(w)}{\partial w}\right) Sw = A\left(\Pi w + \widetilde{\pi}(w)\right) \tag{2.16}$$

$$+ B\left(\Gamma w + \widetilde{g}(w)\right) + f\left(\Pi w + \widetilde{\pi}(w)\right) + Pw.$$

$$\begin{aligned} 0 &= C\pi(w) - Qw \\ &= C\left(\Pi w + \widetilde{\pi}(w)\right) - Qw. \end{aligned}$$

With our choice of Π and Γ satisfying (2.15) the regulator equations in (2.16) become

$$\begin{aligned} \frac{\partial \widetilde{\pi}(w)}{\partial w} Sw &= A\widetilde{\pi}(w) + f\left(\Pi w + \widetilde{\pi}(w)\right) + B\widetilde{\gamma}(w) \\ 0 &= C\widetilde{\pi}(w). \end{aligned} \tag{2.17}$$

In the first equation we solve for $\widetilde{\pi}$ to get

$$\begin{aligned} \widetilde{\pi} &= A^{-1}\left[\frac{\partial \widetilde{\pi}(w)}{\partial w} Sw - f\left(\Pi w + \widetilde{\pi}(w)\right) - B\widetilde{\gamma}(w)\right] \\ 0 &= C\widetilde{\pi}(w). \end{aligned} \tag{2.18}$$

Next we apply the C to the first equation in (2.18) and use the second condition to obtain

$$0 = CA^{-1}\left[\frac{\partial \widetilde{\pi}(w)}{\partial w} Sw - f\left(\Pi w + \widetilde{\pi}(w)\right) - B\widetilde{\gamma}(w)\right].$$

Notice that $G(0) = -CA^{-1}B$ and, under the non-resonance assumption that $G(0)$ is invertible we can solve for $\widetilde{\gamma}$ to obtain

$$\widetilde{g}(w) = G(0)^{-1}CA^{-1}\left[f\left(\Pi w + \widetilde{\pi}(w)\right) - \frac{\partial \widetilde{\pi}(w)}{\partial w} Sw\right]. \tag{2.19}$$

Setting

$$\begin{aligned} F_1\begin{pmatrix} \widetilde{\pi} \\ \widetilde{\gamma} \end{pmatrix} &\equiv A^{-1}\left[\frac{\partial \widetilde{\pi}(w)}{\partial w} Sw - f\left(\Pi w + \widetilde{\pi}(w)\right) - B\widetilde{g}(w)\right]. \\ F_2\begin{pmatrix} \widetilde{\pi} \\ \widetilde{\gamma} \end{pmatrix} &\equiv G(0)^{-1}CA^{-1}\left[f\left(\Pi w + \widetilde{\pi}(w)\right) - \frac{\partial \widetilde{\pi}(w)}{\partial w} Sw\right]. \end{aligned}$$

$$F\begin{pmatrix}\widetilde{\pi}\\ \widetilde{\gamma}\end{pmatrix}=\begin{pmatrix}F_1\\ F_2\end{pmatrix}\begin{pmatrix}\widetilde{\pi}\\ \widetilde{\gamma}\end{pmatrix}.$$

our problem reduces to finding $\mathcal{X}$ so that

$$\mathcal{X}=F(\mathcal{X}),\ \ \mathcal{X}=\begin{pmatrix}\widetilde{\pi}\\ \widetilde{\gamma}\end{pmatrix}. \tag{2.20}$$

This fixed point problem can also be written as

$$\mathcal{F}(\mathcal{X})\equiv\mathcal{X}-F(\mathcal{X})=0.$$

It is in this form that we consider Newton iterations.

2.6 Regulation Methods Using Zero Dynamics Inverse Design

In another research effort we have obtained important insight into the practical solution of regulation problems for boundary control systems using our recently developed zero dynamics inverse design methodology. We have applied this methodology to numerous prototypical examples of linear and nonlinear boundary control systems acting in bounded domains in one and several spatial dimensions. For a wide variety of tracking problems for both interior and boundary control problems the numerical results are astounding. Indeed, the zero dynamics design methodology provides a remarkably simple approach to the design of control laws capable of shaping the response of complicated nonlinear infinite dimensional systems in more mathematically complex case of boundary observation and control. In a most amazing recent discovery we have been able to show that the the zero dynamics design method is intimately related to the geometric design method which is based on center manifold theory in infinite dimensions. Indeed, for certain problems of output regulation the regulator given by the zero dynamics inverse is precisely the regulator equations whose solution provides the desired control law.

One advantage of the zero dynamics inverse (ZDI) design method for designing a feedback compensation scheme achieving asymptotic regulation is that only the value of the signal $y_r(t)$ to be tracked or rejected are known at any instant of time. In analogy with the non-equilibrium formulation of output regulation the control objective is to achieve zero steady-state error together with ultimate boundedness of the state of the system and the controller(s), with a bound determined

by bounds on the norms of the initial data and y_r. In particular, a controller solving this problem depends only on a bound on the norm of y_r not on the particular choice of y_r, which is used only as an input to the controller. ZDI design consists of the interconnection of a stabilizing feedback compensator and a cascade controller, designed in a universal way from the zero dynamics of the closed-loop feedback system. This methodology has evolved over a series of papers for asymptotic regulation for specific linear boundary control systems and for set-point control of linear and nonlinear boundary control systems in one spatial dimension, in which case the input and ontput spaces for the transfer functions were finite dimensional. In this paper, we formulate the main ingredients of the zero dynamics inverse design methodology for a class of abstract linear boundary control systems for problems with infinite dimensional input and output space.

Given a plant to be controlled, the ZDI design consists of the interconnection, via a memoryless filter, of a stabilizing feedback compensator and a cascade controller, designed in a simple universal way from the zero dynamics of the closed-loop feedback system. Key features of the design are to ensure that the closed loop feedback system is "persistently stable" when driven with sufficiently small input signals and that the "zero dynamics inverse" is input-to-state stable (ISS), for appropriate choices of norms. Of course, for nonlinear DPS, the compactness underlying in the success of the ISS philosophy [57], [55] for lumped systems is not directly available in infinite dimensions. In particular, crucial technical details, including the global existence, uniqueness and regularity of solutions to the interconnected systems, need to be checked in order to confirm that the basic design philosophy works. The zero dynamics design methodology we formulate here has evolved over a series of papers for asymptotic regulation for linear boundary control systems and for set-point control of nonlinear boundary control systems [44, 45, 46, 47, 48]. In a paper under review [43] the ZDI philosophy is illustrated by the design of an overall controller solving the problem of asymptotic regulation for a boundary controlled viscous Burgers' equation, for a broad class of input signals.

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CHAPTER IV

PERSONNEL INFORMATION

4.1 Personnel

Christopher I. Byrnes	Professor, Washington University, St. Louis
Alberto Isidori	Professor, Washington University, St. Louis
David S. Gilliam	Professor, Texas Tech University
Anders Lindquist	Professor, KTH, Stockholm, Sweden
Nathan McGregor	Washington University, St. Louis
Brian Whitehead	Washington University

4.2 Honors & Awards

3 IEEE Fellows (Dr.s C.I. Byrnes, A. Isidori, A. Lindquist).

Dr. A. Lindquist has been invited to give an invited one-hour lecture at the joint IEEE/Chinese Control Theory Society Meeting in Shanghai in December 2009

In 2009, Dr. C. I. Byrnes was named an inaugural Fellow of SIAM.

In 2009, Dr. C. I. Byrnes was named an Associate Fellow of AIAA.

In 2009, Dr. C. I. Byrnes held the Giovanni Prodi Chair in Nonlinear Analysis at the University of Wuerzburg, Germany.

Dr. A. Lindquist was awarded the W.T. and Idalia Reid Prize for his contributions to stochastic systems and control at the SIAM Annual Meeting in July 2009.

At the 47th IEEE CDC, Cancun, Mexico, in December 2008, Dr. C. I. Byrnes was awarded the 2008 Hendrik W. Bode Lecture prize from the Control Systems Society of IEEE.

Dr. A. Lindquist presented an invited plenary lecture at the International Congress on the Applications of Mathematics (ICAM), Santiago de Chile, March 13-17, 2006.

Dr. A. Isidori has been appointed President of the International Federation of Automatic Control (IFAC).

Dr. Christopher I. Byrnes was awarded the W.T. and Idalia Reid Prize for his contributions to linear and nonlinear systems and control at the SIAM Annual Meeting in July 2005.

Dr. A. Isidori was installed as Edwin H. Murty Professor of Engineering at Washington University, November 2004.

Dr. A. Isidori was elected Fellow of IFAC, July 2005.

The paper: C. Bonivento, A. Isidori, L. Marconi, A. Paoli, Implicit fault tolerant control: application to induction motors, *Automatica*, 40, pp. 355-371, (2004) was given the triennial *Automatica* award at the IFAC World Congress in Prague, 2005.

At the 42nd IEEE CDC, Maui, Hawaii, in December 2003, C. I. Byrnes, T. Georgiou and A. Lindquist were awarded the 2003 IEEE George S. Axelby Award for the best paper in the IEEE Trans. on Aut. Control.

The paper *A convex optimization approach to the rational covariance extension problem*, by C. I. Byrnes, S. V. Gusev and A. Lindquist was selected in 2000 to be published in an enhanced form in SIAM Review as a "SIGEST" paper.

At the 40th IEEE CDC, Orlando, Florida, in December 2001, A.Isidori was awarded the 2001 Hendrik W. Bode Lecture prize from the Control Systems Society of IEEE.

Te triennial IFAC Best Paper Award, (C.I. Byrnes and A. Isidori), 1993 IFAC World Congress.

IEEE George S. Axelby Award as the best paper in the IEEE Trans. on Aut. Control, 1991 (C.I. Byrnes and A. Isidori).

Dr. C.I. Byrnes was elected in March 2001 as a Foreign Member of the Royal Swedish Academy of Engineering Sciences.

Dr. C.I. Byrnes, was installed as the Edward G. and Florence H. Skinner Professor of Systems and Engineering at Washington University, St. Louis, 1998.

Dr. C.I. Byrnes, elected Fellow of the Academy of Sciences of St. Louis in 1998.

Dr. C.I. Byrnes was awarded an Honorary Doctorate of Technology from the Swedish Royal Institute of Technology, November 1998.

The Graduate College Distinguished Research Award: C.I. Byrnes, 1988, ASU.

Fellow, Japanese Society for the Promotion of Science: C.I. Byrnes, 1986.

"Quazza Medal" awarded to Dr. A. Isidori at 13th IFAC World Congress in San Francisco, 1996 for "Pioneering and Fundamental Contributions to the Design of Nonlinear Feedback Systems."

Alberto Isidori was listed in the Highly-Cited database among the top 10 most-cited authors in Engineering in the world for the period 1981-1999.

Dr. A. Lindquist, Zaborszky Lecturer for the year 2000.

Dr. A. Lindquist, Gordon McKay Visiting Profesor, Berkeley, 2002.

Dr. A. Lindquist, Israel Pollak Distingushed Lecturer, 2005

Dr. A. Lindquist, Foreign Member of Russian Academy of Natural Sciences, 1997.

Dr. A. Lindquist elected Member of the Royal Swedish Acad. of Engr. Sci., 1996.

Dr. A. Lindquist, Honorary Member of Hungarian Operational Res. Soc., 1994.

CHAPTER V

TRANSITIONS AND DISCOVERIES

5.1 AFRL Point of Contact

Dr. Siva S. Banda, Senior Scientist for Control Theory, Air Vehicles Directorate, Air Force Research Laboratory, Wright-Patterson Air Force Base, Ohio. Phone: (937)255-8677, Fax: (937)656-4000, siva.banda@wpafb.af.mil

5.2 New Discoveries

C.I. Byrnes and A. Lindquist, *Method and Apparatus for Speech Analysis and Synthesis*, United States Patent 5,940,791, August 17, 1999.

C.I. Byrnes and A. Lindquist, *Method and Apparatus for Speaker Recognition*, U.S. Patent 6,256,609, July 3, 2001.

C.I. Byrnes, A. Lindquist and T.T. Georgiou, *A Tunable High-Resolution Spectral Estimator*, U.S. Patent 6,400,310, June 4, 2002.

C.I. Byrnes and A. Lindquist, *Method and Apparatus for Speaker Verification Using a Tunable High-resolution Spectral Estimator*, US Patent No. 7,233,898.

12 extensions or foreign patent applications pending.

CHAPTER VI

PUBLICATIONS

1. Edward Allen, John A. Burns, D.S. Gilliam, "On the use of Numerical Methods for Analysis and Control of Nonlinear Convective Systems," *Proceedings 47th IEEE Conference on Decision and Control*, December 2008.

2. C. I. Byrnes, On Brockett's Necessary Condition for Stabilizability and the Topology of Liapunov Functions on R^n, *Communications in Information and Systems*, **8** (2008) 333-352.

3. C. I. Byrnes, Topological Methods for Nonlinear Oscillations, to appear in *The Notices of the Amer. Math. Soc.*

4. C. I. Byrnes and R. W. Brockett, Nonlinear Oscillations and Vector Fields Paired with a Closed One-Form, submitted to *Journal of Differential Equations*.

5. C.I. Byrnes, J. Dockery, D.S. Gilliam, "Bifurcations and Attractors for a Boundary Feedback Controlled Burgers Equation," *Proceedings of European Control Conference* 2007, Kos, Greece, 5037 - 5043.

6. C.I. Byrnes, D.S. Gilliam, "Approximate Solutions of the Regulator Equations for Nonlinear DPS," *Proceedings of the 46th IEEE Conference on Decision & Control*, December 2007.

7. C.I. Byrnes, D.S. Gilliam, "Geometric Output Regulation for a Class of Nonlinear Distributed Parameter Systems," *Proceedings 2008 American Control Conference*, Seattle, Washington, June 2008.

8. C.I. Byrnes, D.S. Gilliam, "The Steady-State Response of a Nonlinear Control System, Lyapunov Stable Attractors, and Forced Oscillations," *Proceedings of the 18th International Symposium on Mathematical Theory of Networks and Systems (MTNS 2008)*, Virginia Tech, Blacksburg, Virginia (USA), 2008.

9. C.I. Byrnes, D.S. Gilliam, Zero dynamics inverse design for asymptotic regulation of the heat equation: the non-colocated case, *Proceedings 2009 American Control Conference*, St. Louis, Mo. (2009).

10. C.I. Byrnes, D.S. Gilliam, "On the transfer function of the zero dynamics for a boundary controlled heat problem in two dimensions," to appear in Proceedings 48th IEEE Conference on Decision and Control, December 2009.

11. C. I. Byrnes, D. S. Gilliam, C.-B. Hu and V. I. Shubov, Asymptotic Regulation for Distributed Parameter Systems, submitted to *Intern. Journal of Robust and Nonlinear Control.*

12. C. I. Byrnes and A. Lindquist, Interior point solutions of variational problems and global inverse function theorems, , *Intern. Journal of Robust and Nonlinear Control*, **16** (2007), 1-18.

13. C. I. Byrnes and A. Lindquist, "Variational problems and global inverse function theorems," *Intern. Journal of Robust and Nonlinear Control* **16** (2007) 1 -18.

14. C. I. Byrnes and A. Lindquist, A note on the Jacobian Conjecture , *The Proc. of the AMS*, **136** (2008) 3007-3011.

15. C. I. Byrnes and A. Lindquist, Important Moments in Systems and Control , *SIAM J. Control and Optimization*, **47** (2008) 2458-2469.

16. C. I. Byrnes and A. Lindquist, Important moments in systems, control and optimization, Proc. 48th IEEE Conf. Decision and Control, 2009, to be published.

17. C. I. Byrnes and A. Lindquist, The moment problem for rational measures: convexity in the spirit of Krein, in Modern Analysis and Application: To the centenary of Mark Krein, Vol. I: Operator Theory and Related Topics, BirkhŁuser, 2009.

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14. ABSTRACT
The principal goal of this research program was to develop a systematic methodology for the design of feedback control schemes capable of shaping the response of complex dynamical systems. A continuation of an ongoing research effort, the specific research program we proposed was aimed at the development of a systematic control methodology for lumped and distributed parameter systems, applicable to both the equilibrium and the nonequilibrium cases. The typical design objectives involve designing feedback schemes which achieve one or more of the following: asymptotic tracking, an appropriate form of internal stabilization, and asymptotic disturbance rejection.

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